Int. J. Heat Mass Transfer. Vol. 24, No. 7, pp. 1229–1236, 1981
Printed in Great Britain

0017-9310/81/071229-08 $02.00/0
Pergamon Press Ltd.

LUMPED HEAT-TRANSFER COEFFICIENTS FOR THERMAL REGENERATORS

CLARE HINCHCLIFFE and A. J. WILLMOTT
Department of Computer Science, University of York, Heslington, York, U.K.

(*Received* 9 *April* 1980 *and in revised form* 20 *February* 1981)

Abstract—Traditional methods of representing the resistance to heat transfer within the packing of a thermal regenerator have involved the use of lumped or bulk heat-transfer coefficients in which this resistance is added to the resistance between gas and solid at the surface of the packing. This paper describes how the chronological variation of this latitudinal solid conduction effect can be embodied within such lumped heat-transfer coefficients.

NOMENCLATURE

A_2, maximum exit gas temperature calculated using 2-D method [K];
A_3, maximum exit gas temperature calculated using 3-D method [K];
B_2, minimum exit gas temperature calculated using 2-D method [K];
B_3, minimum exit gas temperature calculated using 3-D method [K];
Bi, Biot modulus;
C, specific heat [J kg^{-1} K^{-1}];
d, thickness of packing material of regenerators [m];
h, surface heat-transfer coefficient [W m^{-2} K^{-1}];
$\bar{h}$, lumped heat-transfer coefficient [W m^{-2} K^{-1}];
k, thermal conductivity of packing material of regenerator [W m^{-1} K^{-1}];
L, length of regenerator [m];
$\dot{m}_f$, mass flowrate of gas [kg s^{-1}];
M_m, mass of regenerator packing [kg];
P, period length [s];
S, surface area available for heat transfer in regenerator packing [m^2];
T_m, mean solid temperature [K];
T_0, surface solid temperature [K];
t, gas temperature [K];
W, dimensionless time (3-D model);
x, distance perpendicular to gas flow into regenerator packing from solid surface [m];
y, distance along the length of the regenerator [m];
Z, dimensionless distance corresponds to x;
n, parameter characterising packing geometry;

Greek symbols

α, thermal diffusivity of regenerator packing material [m^2 s^{-1}];
ξ, dimensionless distance corresponding to y (3-D model);
ζ, dimensionless distance corresponding to y (2-D model);
η, dimensionless time (2-D model);
ϕ_H, Hausen correction factor;
$\phi(\tau)$, time varying correction factor;
τ, time [s];
ψ, temperature ratio $(A_2 - B_2)/(A_3 - B_3)$;
Λ, reduced length (3-D model);
$\bar{\Lambda}$, reduced length (2-D model);
$\bar{\Pi}$, reduced period (2-D model);
Ω, reduced time (3-D model).

Superscripts

$'$, refers to hot period;
$''$, refers to cold period.

Subscripts

f, refers to gas;
m, refers to solid.

INTRODUCTION

HAUSEN [1] developed a bulk heat-transfer coefficient $\bar{h}$ of the form

$$\frac{1}{\bar{h}} = \frac{1}{h} + \frac{d}{2(n+2)k}\phi_H \tag{1}$$

where $n = 1$ for slabs (plane walls) of thickness d; $n = 2$ for solid cylinders of diameter d; and $n = 3$ for spheres of diameter d. This incorporates the surface heat-transfer coefficient h and the resistance to heat transfer within the regenerator packing, as represented by the $d\phi_H/[2(n+2)k]$ term. This coefficient can be used in the conventional model of the thermal performance of the regenerator, set out in the differential equations

$$\bar{h}S(T_m - t) = \dot{m}_f C_f L \frac{\partial t}{\partial y} \tag{2}$$

$$\bar{h}S(t - T_m) = M_m C_m \frac{\partial T_m}{\partial \tau} \tag{3}$$

and discussed by many authors including Willmott [2]. At any position y in the regenerator, the solid temperature T_m used in these differential equations is the local mean temperature defined by equation (4)

$$T_m(y,\tau) = \frac{1}{d}\int_0^d T(x,y,\tau)\,dx \quad (4)$$

where x is the direction into the wall of the packing material, perpendicular to the gas flow direction y. The advantage of this approach is that nonlinear features can be embodied into a regenerator model without calculating explicitly the internal chronological variations of solid temperature in the x-direction. In particular Willmott [3] and more recently Razelos and Benjamin [4] have used the bulk heat-transfer coefficient $\bar{h}$ as defined by equation (1) to represent regenerator performance under circumstances where the gas flowrate $\dot{m}_f(\tau)$ varies continuously with time in the cold period of regenerator operation. The same bulk heat-transfer coefficient can be used in other nonlinear models of regenerators including those embodying temperature dependent gas specific heat or radiant as well as convective heat transfer between gas and solid, for example. The importance of this possible use of equations (2) and (3) for nonlinear models justifies a re-examination of the assumptions built into equation (1) and the development of improved coefficients.

However the deficiency of this approach as it presently stands is that the Hausen ϕ_H is a blanket factor applied as a constant throughout the duration of both the hot and cold periods. It will be seen shortly that it is computed as a time mean, averaging out any local chronological variations in the resistance to heat transfer within the packing wall. It turns out therefore if the exit gas temperature is computed using the Hausen ϕ_H model (the so-called 2-D calculation method), there is a significant difference, see Fig. 2, between it and the exit gas temperature computing by the 3-D calculation method in which the unsteady state heat conduction within the wall of the packing is embodied explicitly within the mathematical model. The purpose of this paper is to describe the basis of a method whereby the 2-D calculation procedure is modified, by introducing a time varying $\phi(\tau)$ factor, thereby enabling a closer correlation between the exit gas temperatures computed by the 2-D and 3-D methods of computation.

IDEALIZATION OF REGENERATOR OPERATION

Bahnke and Howard [5], Tipler [6], Willmott [7] and Razelos and Benjamin [4] all regard the effect of longitudinal (parallel to gas flow) heat conduction in the solid as negligible for most practical regenerators. This paper restricts consideration to the representation of latitudinal solid conduction in a direction perpendicular to gas flow.

Although in the long term, it is intended that any improved bulk heat-transfer coefficients can be built into nonlinear models with comparative ease, in this study it is assumed that the thermal properties of both gas and solid, the gas entrance temperature and the gas flowrate do not vary with time within a period, although they may be different in the hot and cold periods.

Implicit in the definition of the mean solid temperature $T_m(y, \tau)$ in equation (4) is that the regenerator packing consists of parallel walls, cylindrical rods or spherical balls of heat storing material. The characteristic dimension of the packing is d [see equation (1)] while the depth of the packing is L. While consideration is restricted here to slabs, solid cylinders and spheres, it should be noted that Razelos and Lazaridis [8] considered a chequerwork of a hollow cylindrical geometry. Hausen initially considered the slab [1] but went on to deal with cylindrical and spherical forms of packing.

DEVELOPMENT OF BULK HEAT-TRANSFER COEFFICIENTS

In the middle of the regenerator, the solid temperature varies linearly with time in both the hot and the cold period. This can be regarded as the consequence of there being an unchanging heat flux in each period of regenerator operation at the surface of the solid at the position in the middle of the regenerator under consideration. Although the solid temperature varies nonlinearly with time close to the entrances of the regenerator as a consequence of there being an unchanging inlet gas temperature, and this is discussed extensively by Hausen [1], nevertheless it is assumed that the bulk heat-transfer coefficient developed on the basis of a constant heat flux at the solid surface in each period of regenerator operation, can be applied at all positions in the regenerator. Butterfield, Schofield and Young [10] discuss this problem and show the assumption is acceptable in most cases for the form of lumped heat-transfer coefficient developed by Hausen.

The lumped heat-transfer coefficient $\bar{h}$ is associated with the mean solid temperature T_m. It is related to the surface heat-transfer coefficient h and surface solid temperature T_0 in such a way that the heat flux at any instant τ computed using $\bar{h}$ is the same as that calculated using h. This is expressed by equation (5)

$$\bar{h}(\tau)[t(\tau) - T_m(\tau)] = h[t(\tau) - T_0(\tau)] \quad (5)$$

or

$$\frac{h}{\bar{h}(\tau)} = \frac{t(\tau) - T_m(\tau)}{t(\tau) - T_0(\tau)}. \quad (6)$$

Hausen's form of the lumped heat-transfer coefficient in equation (1) can be re-written

$$\frac{h}{\bar{h}(\tau)} = 1 + \frac{Bi}{(n+2)}\phi(\tau). \quad (7)$$

Note that $Bi = h\,d/2k$, the Biot modulus. Equating the right-hand sides of (6) and (7)

$$\phi(\tau) = \frac{n+2}{Bi}\left\{\frac{T_0(\tau) - T_m(\tau)}{t(\tau) - T_0(\tau)}\right\}. \quad (8)$$

The unsteady-state conduction of heat in the walls comprising the regenerator chequerwork in a direction perpendicular to gas flow is represented by equation (9)

$$\frac{\partial T}{\partial W} = \frac{1}{Z^{n-1}} \frac{\partial}{\partial Z}\left(Z^{n-1}\frac{\partial T}{\partial Z}\right) \tag{9}$$

where the dimensionless time $W = 4\alpha\tau/d^2$ and the dimensionless distance $Z = 2x/d$. In the middle of the packing, there is zero heat flux from considerations of symmetry

$$\left.\frac{\partial T}{\partial Z}\right|_{Z=1} = 0. \tag{10}$$

At the surface of the solid, the heat transfer between gas and solid is represented by equation (11)

$$\left.\frac{\partial T}{\partial Z}\right|_{Z=0} = Bi[T_0(\tau) - t(\tau)]. \tag{11}$$

To evaluate the $\phi(\tau)$ factor on the basis of a constant heat flux at the solid surface in each period of regenerator operation, it is necessary to solve equation (9) for cyclic equilibrium with boundary condition (10) and with the surface heat flux

$$Bi[T_0(\tau) - t(\tau)] = \pm q \text{ (constant)} \tag{12}$$

for equal periods more generally $-q$ for the hot period, $+q\Omega'/\Omega''$ for the cold period.

Recently Kern [11] presented an analytical solution to this equation (11) for $n = 1$ for square wave gas temperatures rather than square wave heat flux at the solid surface. This analytical approach is tedious and leads to rather lengthy expressions for solid temperature, which are important in the development of $\phi(\tau)$ in equation (8). Kern mentions the possible need to sum the infinite series developed for up to 8000 terms in certain cases. In this paper these problems are avoided by solving equation (9) numerically.

At cyclic equilibrium, any local value of $\phi(\tau)$ obtained from equation (8) is independent of the size of the temperature difference between gas and solid surface $[t(\tau) - T_0(\tau)]$ from the linearity of this expression (8). It follows that for a constant heat flux $Bi[t(\tau) - T_0(\tau)]$ in the hot and cold periods of regenerator operation, $\phi(\tau)$ is independent of Biot modulus. The constant heat flux can be made up of a large temperature difference and small Biot modulus or vice versa: in either case, the value of $\phi(\tau)$ at any instant remains unchanged.

The form of the $\phi(\tau)$ is thus a function only of the dimensionless period lengths Ω' and Ω'' (where $\Omega' = 4\alpha P'/d^2$ and $\Omega'' = 4\alpha P''/d^2$).

However, the factor $\phi(\tau)$ becomes a function also of Biot modulus for cases corresponding to a time varying surface heat flux. Butterfield [10] has noted this but has concluded that for practical situations, the overall effect of Bi can be neglected.

For the square wave heat flux, the established parabolic temperature profile at the end of the cold/hot period is completely distorted at the start of the subsequent hot/cold period (see Fig. 1) and remains so until the new parabolic profile is formed. The shorter the period lengths Ω' and Ω'', the more severe the overall effect of these inversions of the parabolic profile.

Hausen's approach [1] to this problem was to take a chronological mean ϕ_H of $\phi(\tau)$ over the whole cycle of regenerator operation, based on a square wave heat flux

$$\phi_H = \frac{1}{P' + P''}\int_0^{P'+P''} \phi(\tau)\,d\tau$$

or

$$\phi_H = \frac{1}{\Omega' + \Omega''}\int_0^{\Omega'+\Omega''} \phi(W)\,dW \tag{13}$$

if dimensionless time W is employed.

This mean value ϕ_H was built into the lumped heat-transfer coefficient, using equation (1), the same mean value being applied to both hot and cold periods.

Hausen developed a simple form for the mean value ϕ_H, namely

$$\phi_H = 1 - \frac{1}{(n+3)^2 - 1}\left\{\frac{1}{\Omega'}\ \frac{1}{\Omega''}\right\} \tag{14}$$

for

$$\frac{1}{\Omega'} + \frac{1}{\Omega''} \leqslant 5(n+1)/2.$$

For other values of $1/\Omega' + 1/\Omega''$

$$\phi_H = \pi(n+2)\bigg/\sqrt{\left(\varepsilon + 18\left\{\frac{1}{\Omega'} + \frac{1}{\Omega''}\right\}\right)} \tag{15}$$

where $\varepsilon = 2.7$ for plates; $\varepsilon = 9.9$ for cylinders; and $\varepsilon = 27.0$ for spheres.

The use of the time mean (13) averages out over the whole cycle and therefore conceals the local (in time) effect of the inversions of the parabolic profile upon regenerator performance. Willmott [7] examined this problem in detail for slabs by comparing the solution of the differential equations (2) and (3) using the lumped heat-transfer coefficient $\bar{h}$ with the solution computed using the more complete model in which the internal heat diffusion within the solid is represented explicitly using the diffusion equation (9) with boundary conditions (10) and (11); the solid and gas temperatures down the length of the regenerator are related by the dimensionless form of equation (2), (but using h and T_0 instead of $\bar{h}$ and T_m), namely

$$\frac{\partial t}{\partial \xi} = T_0 - t \tag{16}$$

where $\xi = hSy/m_f c_f L$. Equations (9), (10), (13) and (16) comprise what is called the 3-D model. Equations (2) and (3) can be reduced to similar dimensionless form

$$\frac{\partial t}{\partial \zeta} = T_m - t \tag{17}$$

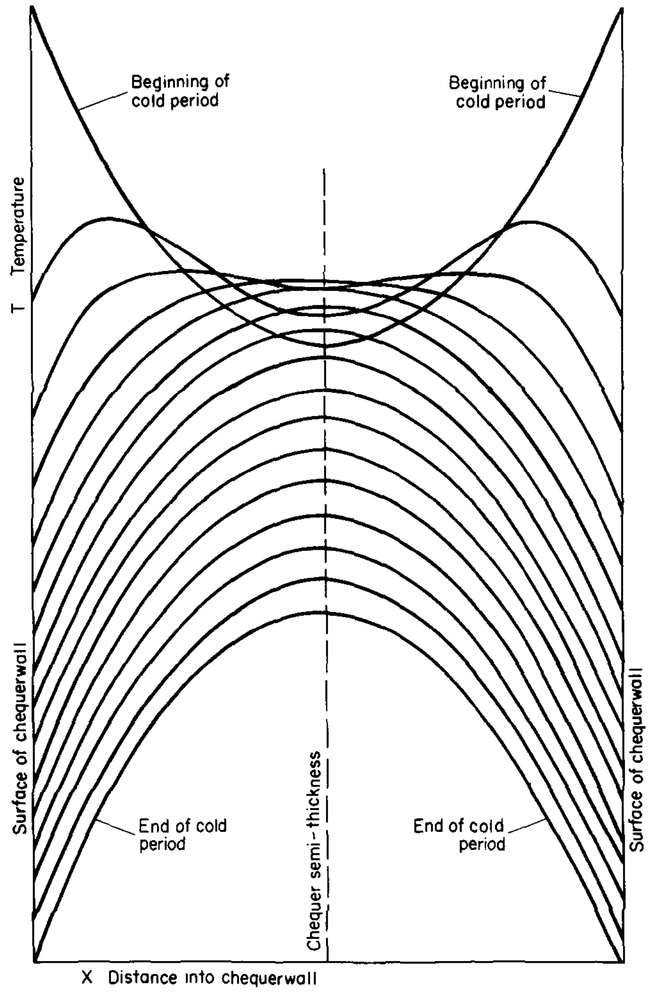

FIG. 1. Solid temperature profile changes as a period of operation progresses.

$$\frac{\partial T_m}{\partial \eta} = t - T_m \tag{18}$$

where $\zeta = \bar{h}Sy/\dot{m}_f C_f L$ and $\eta = \bar{h}S\tau/\dot{m}_m C_m$ and these equations comprise the 2-D model.

The configuration and operation of the regenerator can be summarized by a set of dimensionless parameters set out in Table 1.

Willmott [7] compared the two idealizations for slabs by calculating the chronological variation of exit gas temperature using both the 3-D and 2-D models for the same range of regenerator configurations and operating conditions. A typical comparison for the cold period is displayed in Fig. 2. The 2-D model predicts a linear variation of exit gas temperature whereas the 3-D model forecasts an initially higher but sharply declining exit temperature, at a time when the solid temperature parabolic profile is suffering an inversion. Subsequently this exit temperature becomes

Table 1. Regenerator dimensionless parameters

3-D model	Hot period	Cold period
Reduced length	$\Lambda' = \dfrac{h'S}{\dot{m}'_f C'_f}$	$\Lambda'' = \dfrac{h''S}{\dot{m}''_f C''_f}$
Reduced time	$\Omega' = \dfrac{4\alpha P'}{d^2}$	$\Omega'' = \dfrac{4\alpha P''}{d^2}$
Biot modulus	$Bi' = \dfrac{h'd}{2k}$	$Bi'' = \dfrac{h''d}{2k}$
2-D model ϕ_H factor		
Reduced length	$\bar{\Lambda}' = \dfrac{\bar{h}'S}{\dot{m}'_f C'_f}$	$\bar{\Lambda}'' = \dfrac{\bar{h}''S}{\dot{m}''_f C''_f}$
Reduced period	$\bar{\Pi}' = \dfrac{\bar{h}'SP'}{M_m C_m}$	$\bar{\Pi}'' = \dfrac{\bar{h}''SP''}{M_m C_m}$

linear in variation with approximately the same gradient as that predicted for the 2-D model; this corresponds to the time when the parabolic profile is well established, remaining so until the end of the period under consideration. The difference in the model predictions was parameterized using

$$\psi = \frac{A_2 - B_2}{A_3 - B_3} \tag{19}$$

(see Fig. 3). The smaller the value of ψ, the sharper the effect of the reversals upon the acceptability of the Hausen assumption of being able to use an average ϕ_H factor. Indeed, the shorter the cyclic times Ω' and the smaller the ϕ_H factor [see equations (14) and (15)] and the smaller the ψ parameter, and hence the poorer the comparison between the 2-D and 3-D models.

IMPROVED LUMPED HEAT-TRANSFER COEFFICIENTS

The kernel of the proposals in this paper lies in the representing of inversion of the solid temperature profile within the lumped heat-transfer coefficient for the 2-D model, thereby avoiding use of the possibly computationally uneconomic 3-D model. This is achieved by retaining the factor $\phi(\tau)$ as a function of time using equation (18) directly. This $\phi(\tau)$ is computed for a square wave heat flux $Bi\ [t(\tau) - T_0(\tau)\}$. However, the inversion of the parabolic profile after each reversal is manifest explicitly in the time variation of $T_0(\tau) - T_m(\tau)$ and hence of $\phi(\tau)$. The lumped heat-transfer coefficient represents this time variation using equation (17). The values of $\bar{\Lambda}'$, $\bar{\Lambda}''$, $\bar{\Pi}'$, $\bar{\Pi}''$ change continuously, following the time variations of $\phi(\tau)$.

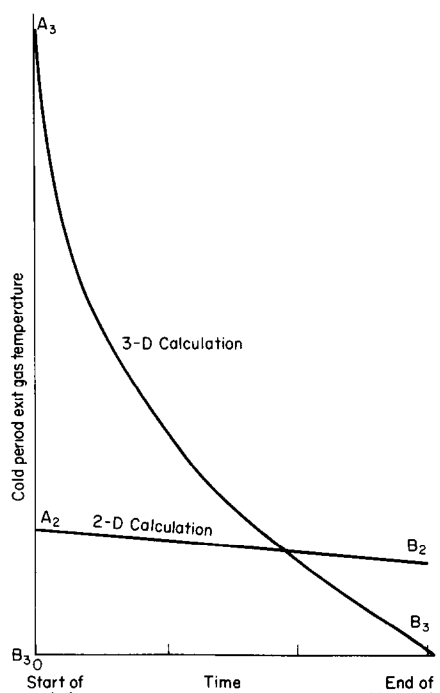

FIG. 2. Comparison between exit gas temperatures computed using 3-D equations and using 2-D equations.

The method of calculation we propose is in two stages. Firstly, the variation of $\phi(\tau)$ for the hot and cold periods is computed by solving numerically equations (9), (10) and (11) for cyclic equilibrium, for a square wave heat flux with the values of Ω' and Ω'' particular to the regenerator operation under consideration. While this takes no advantage of any possible analytical solution to this problem, it allows all possible combinations of values of Ω', Ω'', to be dealt with directly. This implies possibly different forms of $\phi(\tau)$ for the hot and cold periods. The second stage of the calculation of regenerator performance consists of a conventional solution of the 2-D equations (2) and (3) using the time varying $\phi(\tau)$ factor. At this second stage, it is implied that equations (2) and (3) may also embody nonlinear features such as the temperature dependence of gas and solid thermal properties or the time variation of gas flowrate.

FORM OF THE TIME VARYING $\phi(\tau)$ FACTOR

At the start of a period, the internal temperature profile of the solid is that remaining from the end of the previous period so that at the start of the hot period, for example, $t(\tau) - T_0(\tau) > 0$ but $T_0(\tau) - T_m(\tau) < 0$. As the period progresses, the solid temperature profile is first distorted and then becomes inverted when $T_0(\tau) - T_m(\tau) > 0$. In the symmetric case, the absolute value of $T_0(\tau) - T_m(\tau)$ at the start of a period is equal to that at the end of the period at cyclic equilibrium. If $\phi(0)$ is the value of the factor at the start of the period and $\phi(\Omega)$ is the value of the end of a period, then it follows from equation (8) that for the symmetric case

$$\phi(0) = -\phi(\Omega). \tag{20}$$

In the general case, the factor $\phi(\tau)$ varies within the range -1 and $+1$ as the period under consideration progresses.

Displayed in Fig. 4 are graphical representations of typical examples of the chronological variation of $\phi(\tau)$ for $\Omega = 1.3333$ (the corresponding value of ϕ_H is 0.9) and $\Omega = 0.2216$ ($\phi_H = 0.5$). It can be seen that at the start of a period $\phi(\tau)$ increases rapidly as the internal solid temperature profile is inverted. Once this inversion is completed, $\phi(\tau)$ settles to a fixed value.

APPLICATION OF THE TIME VARYING $\phi(\tau)$ FACTOR

We restrict our considerations to slab geometry and to the symmetric case in this paper. In a later paper it is proposed to examine the use of the $\phi(\tau)$ for unbalanced regenerators, and to other geometries.

Presented here are the values of ψ computed with $\phi(\tau)$ for $\phi_H = 0.9$ and $\phi_H = 0.8$ and again $\bar{\Pi} = 1$. These are shown in Tables 2 and 3.

Both the ϕ_H and $\phi(\tau)$ models compute similar values for the thermal ratio; the values are within 0.005 of one another over the range of parameters presented in Tables 2 and 3. This clearly vindicates the Hausen approach to the problem in particular the use of the ϕ_H

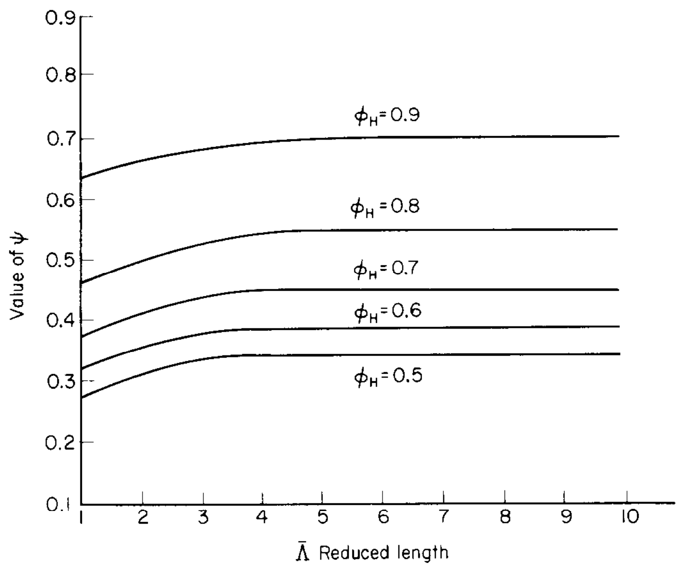

FIG. 3. Variation of ψ with respect to reduced length Λ and the Hausen ϕ_H factor.

$$\psi = \frac{(\text{max exit gas temp} - \text{min exit gas temp})\,\text{2-D}}{(\text{max exit gas temp} - \text{min exit gas temp})\,\text{3-D}} \quad \text{for } \bar{\Pi} = 1.$$

Table 2. Value of the ratio $\psi = (A_2 - B_2/A_3 - B_3)$ for $\phi_H = 0.9$ and $\bar{\Pi} = 1$

Reduced length $\bar{\Lambda}$	ϕ_H model values	$\phi(\tau)$ model values
1	0.6484	1.1864
2	0.6726	1.0859
4	0.6926	1.0445
8	0.6983	1.0433
10	0.6990	1.0438

Table 3. Value of the ratio ψ for $\phi_H = 0.8$ and $\bar{\Pi} = 1$

Reduced length $\bar{\Lambda}$	ϕ_H model values	$\phi(\tau)$ model values
1	0.4671	1.3137
2	0.5085	1.1217
4	0.5375	1.0870
8	0.5454	1.0922
10	0.5469	1.0923

parameter for the calculation of time average temperature behaviour.

The development of the $\phi(\tau)$ model assumes that the square wave heat flux at the solid surface, typical of the thermal behaviour on the middle of a regenerator is applicable at all positions. It is well known that the constant entrance gas temperature gives rise to an exponentially decreasing (with time) heat flux at this entrance; the importance of this nonlinear behaviour increases the shorter the regenerator. Thus the value of ψ in general decreases with increasing reduced length $\bar{\Lambda}$ for the $\phi(\tau)$ model. Even in the worse case ($\bar{\Lambda} = 1$) presented, the value of ψ is closer to the ideal value of 1 for the $\phi(\tau)$ model than with the ϕ_H Hausen model.

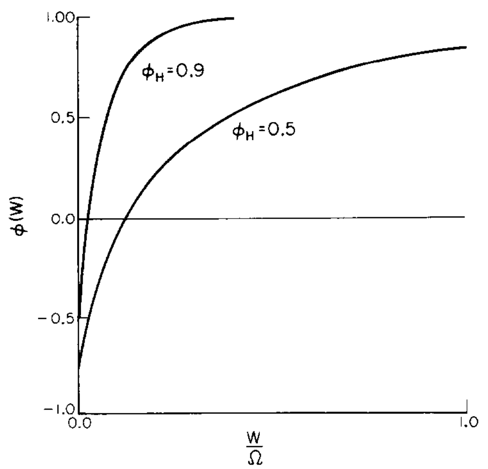

FIG. 4. Chronological variation of $\phi(W)$ for $\phi_H = 0.9$ and $\phi_H = 0.5$ (Symmetric case $\Omega = \Omega' = \Omega''$).

LIMITATIONS OF NEW METHOD

Equation (9) can be re-written as

$$\frac{1}{\bar{h}(\tau)} = \frac{1}{h}\left[1 + \frac{Bi\phi(\tau)}{n+2}\right]. \tag{21}$$

Now $\phi(\tau)$ is computed as a function of Ω alone and, as has been discussed, $\phi(\tau)$ lies in the range $[-1, 1]$. It follows that if, for any stage of the calculation, $\phi(\tau) < 0$ and $|Bi\phi(\tau)| > n + 2$, a negative value of $\bar{h}(\tau)$ may be computed. It turns out therefore, since the limiting values of $\phi(\tau)$ for the start of a period are close to -1 for many typical regenerator configurations, this method should not be used in general for values of Biot modulus $Bi > n + 2$.

Should the $\phi(\tau)$ model be used with too high values of Bi the regenerator simulation becomes completely unstable and extremely large positive and negative temperatures are computed.

Provided this limitation is observed, the $\phi(\tau)$ method can be used with confidence. It might be argued that the method should not be used for small reduced lengths $\bar{\Lambda}$. The choice then to be made is between the full 3-D method and the $\phi(\tau)$ method and the user of the computer simulation of the regenerator must decide what errors, measured perhaps in terms of ψ, can be accepted relative to available experimental accuracy. However, it is certain that ψ is always closer to unity for the $\phi(\tau)$ method than the ϕ_H method for all Λ provided $Bi < n + 2$.

THE HEGGS AND CARPENTER APPROACH

In a recent paper, Heggs and Carpenter [12] developed what they called a "Modified Infinite-conduction System". As in the work presented here, Heggs and Carpenter recognized that if the 2-D model was to be used to approximate the regenerator performance predicted by the 3-D model, a time varying heat transfer coefficient should be introduced. Whereas we propose the use of the time varying $\phi(\tau)$ factor to generate a time varying lumped heat-transfer coefficient $\bar{h}(\tau)$, equation (21), Heggs and Carpenter chose to modify the surface heat-transfer coefficient by a multiplication factor $\varepsilon(\tau)$, that is $\bar{h}(\tau) = h\varepsilon(\tau)$.

Both approaches are equivalent if the method of determining $\phi(\tau)$, explicitly in our method, indirectly in the method of Heggs and Carpenter, is the same. This can be expressed for the plain wall in the following way

$$\frac{1}{\bar{h}(\tau)} = \frac{1}{h\varepsilon(\tau)} = \frac{1}{h} + \frac{d\phi(\tau)}{3\lambda}$$

or

$$\frac{1}{\varepsilon(\tau)} = 1 + \frac{Bi}{3}\phi(\tau). \qquad (22)$$

This equation (22) highlights what must be the limitation of both approaches. If at any instant $\phi(\tau) = -3/Bi$, then $\varepsilon(\tau)$ becomes infinitely large. In such a case, the modified 2-D simulation will become completely unstable.

In so far as the $\varepsilon(\tau)$ treatment of the problem helps to throw light on the potential instability of the methodology for large values of Biot modulus, it is particularly useful. However Heggs and Carpenter do not develop the $\phi(\tau)$ factor as a function of Ω' and Ω'' alone and do not solve the diffusion equation (9) with a square wave heat flux [equation (12)] boundary condition. Instead they solve equation (9) for a gas temperature which varies linearly with time, with $\partial t/\partial\tau = R_t$ in the hot period and $\partial t/\partial\tau = -R_t$ in the cold period. Appropriate values of R_t are presented graphically having been estimated from a whole series of computational experiments using the 3-D model.

The development of the R_t factor is based on symmetric regenerators although it is suggested that an average value of R_t can be used for unbalanced regenerators using the hot and cold parameters separately.

The chief disadvantage of the Heggs and Carpenter approach lies in the fact that appropriate values of R_t must be interpolated from a set of graphs since R_t is a function of the three dimensionless parameters embodied in the 3-D model for the symmetric case.

In the method proposed here, the time variation of $\phi(\tau)$ can be computed for any combination of Ω' and Ω'' and quite independently of any other parameters.

However, the overriding consideration in favour of both the $\phi(\tau)$ and $\varepsilon(\tau)$ approaches is that it is often quite impracticable to compute regenerator performance using a 3-D model embodying nonlinear features such as temperature dependent specific heat or radiative heat-transfer coefficients. The introduction of the $\phi(\tau)$ approach adds little in the way of complications to 2-D models embodying such nonlinear features.

It is in this area that we believe the strength of our proposals lies and we hope other workers in the field will be able to exploit the $\phi(\tau)$ method for the particular nonlinear regenerator models they are required to examine.

Acknowledgements—We must thank the Science Research Council who supported Clare Hinchcliffe by means of a research studentship during the period when this work was carried out. We thank the University of York Computing Service for the running of our computer programs. We are endebted to Valerie Fry and Sara Bain for the typing of the manuscript.

REFERENCES

1. H. Hausen, Vervollständigte Berechnung des Wärmeaustausches in Regeneratoren, *Z. VDI-Beiheft Verfahrenstechnik* No. 2, 31–43 (1942).
2. A. J. Willmott, Digital computer simulation of a thermal regenerator, *Int. J. Heat Mass Transfer* **7**, 1291–1302 (1964).
3. A. J. Willmott, Simulation of a thermal regenerator under conditions of variable mass flow, *Int. J. Heat Mass Transfer* **11**, 1105–1116 (1968).
4. P. Razelos and M. K. Benjamin, Computer model of thermal regenerators with variable mass flow rates, *Int. J. Heat Mass Transfer* **21**, 735–743 (1978).
5. G. D. Bahnke and C. P. Howard, The effect of longitudinal heat conduction on periodic-flow heat exchanger performance, *J. Engng Pwr.* 105–120 (1964).
6. E. Tipler, A simple theory of the heat regenerator, Shell Technical Report, ICT/14 (1947).
7. A. J. Willmott, The regenerative heat exchanger computer representation, *Int. J. Heat Mass Transfer* **12**, 997–1014 (1969).
8. P. Razelos and A. Lazaridis, A lumped heat transfer coefficient for periodically heated hollow cylinders, *Int. J. Heat Mass Transfer* **10**, 1373–1387 (1967).
9. H. Hausen, *Wärmeübertragung im Gegenstrom, Gleichstrom und Kreuzstrom*, 2nd edn. Springer Verlag, Berlin (1976).
10. P. Butterfield, J. S. Schofield and P. A. Young, Hot blast stoves, *J. Iron Steel Inst.* **201**, 497–508 (1963).

11. J. Kern, On regenerative heat exchange with realistic boundary conditions, *Proc. 6th International Heat Transfer Conference* **4**, 315–320 (1978).
12. P. J. Heggs and K. J. Carpenter, A modification of the thermal regenerator infinite conduction model to predict the effects of intraconduction, *Trans. I. Chem. E.* **57**, 228–236 (1979).

COEFFICIENTS GLOBAUX DE TRANSFERT THERMIQUE DES REGENERATEURS DE CHALEUR

Résumé—Des méthodes traditionnelles de représentation de la résistance au transfert thermique dans l'assemblage d'un régénérateur de chaleur ont conduit à l'usage de coefficients globaux de transfert thermique dans lesquels la résistance est ajoutée à la résistance entre gaz et solide à l'interface. Cet article décrit comment la variation temporelle de l'effet de conduction dans le solide peut être incorporée dans ces coefficients globaux.

ZUSAMMENGEFASSTE WÄRMEÜBERTRAGUNGSKOEFFIZIENTEN FÜR THERMISCHE REGENERATOREN

Zusammenfassung—Traditionelle Methoden zur Darstellung des Wärmeübertragungswiderstandes in der Speichermasse eines thermischen Regenerators haben von zusammengefaßten oder mittleren Wärmeübertragungskoeffizienten Gebrauch gemacht, bei denen dieser Widerstand zum Übergangswiderstand zwischen Gas und Feststoff an der Oberfläche der Speichermasse hinzugenommen wird. In dieser Arbeit wird beschrieben, wie die zeitliche Änderung dieses Feststoff-Querleitungseffektes in solchen zusammengefaßten Wärmeübertragungskoeffizienten berücksichtigt werden kann.

ЭФФЕКТИВНЫЕ КОЭФФИЦИЕНТЫ ТЕПЛОПЕРЕНОСА В ТЕПЛОВЫХ РЕГЕНЕРАТОРАХ

Аннотация — Традиционные методы представления термического сопротивления в насадке теплового регенератора требуют использования эффективных или объемных коэффициентов теплопереноса, в которых это сопротивление суммируется с термическим сопротивлением между газом и твердым телом на поверхности насадки. Показано, каким образом в эффективных коэффициентах теплопереноса может учитываться изменение во времени поперечной передачи тепла теплопроводностью в твердом теле.